PROBLEMS.

44. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.

Find the general term in the series 1, 3, 10, 35, 126, 462, 1716, 6435, 24310, ...,which plays a remarkable part in some recent theorems in my theory of Regular Polygons.

45. Proposed by WILLIAM HOOVER, A. M., Ph. D., Ohio State University, Athens, Ohio.

Find x from $\cos^{-1}\dfrac{1-x^2}{1+x^2}+\tan^{-1}\dfrac{2x}{1-x^2}=\dfrac{4\pi}{3}$.

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

32. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio State University, Athens, Ohio.

If a conic be incribed in a triangle and its focus move along a given straight line, the locus of the other focus is a conic circumscribing the triangle.

I. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Using trilinear co-ordinates the equation to the inscribed ellipse is of the form $\sqrt{(l\alpha)}+\sqrt{(m\beta)}+\sqrt{(n\gamma)}=0$.

Let α' β' γ', be the co-ordinates of the one focus, then

$\dfrac{\alpha}{\alpha'}=\dfrac{\beta}{\beta'}$, $\dfrac{\beta}{\beta'}=\dfrac{\gamma}{\gamma'}=\dfrac{\alpha}{\alpha'}$ are the equations to the lines joining it to the vertices of the triangle. The lines to the other focus make equal angles with the sides of the triangles, hence, their equations are $\alpha'\alpha=\beta'\beta$, $\beta'\beta=\gamma'\gamma$, $\gamma'\gamma=\alpha'\alpha$. $\therefore$ the co-ordinates of the other focus may be taken

$\dfrac{1}{\alpha'}$, $\dfrac{1}{\beta'}$, $\dfrac{1}{\gamma}$; from this relation, if we are given the equation of any locus described by one focus, we can at once write down the equation of the locus described by the other focus.

$\therefore$ If the first focus describes the straight line $l\alpha+m\beta+n\gamma=0$, the second will describe the locus whose equation is

$$\frac{l}{\alpha}+\frac{m}{\beta}+\frac{n}{\gamma}=0,$$ a conic circumscribing the triangle.

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio State University, Athens, Ohio.

In trilinear co-ordinates, let the foci be $(\alpha',\beta',\gamma'),(\alpha'',\beta'',\gamma'')$. Then since the product of the perpendiculars from the foci upon tangents to a conic is constant, we should have $\alpha'\alpha''=\beta'\beta''=\gamma'\gamma''=k\ldots.(1)$.

If $l\alpha+m\beta+n\gamma=0\ldots.(2)$ be the locus of (α',β',γ'), it is plain from (1) that $\dfrac{l}{\alpha''}+\dfrac{m}{\beta''}+\dfrac{n}{\gamma''}=0\ldots.(3)$, or $l\beta\gamma+m\alpha\gamma+n\alpha\beta=0\ \ldots(4)$, by dropping accents, which is a circumscribing conic.

33. Proposed by Professor B. F. SINE, Shenandoah Normal College, Reliance, Virginia.

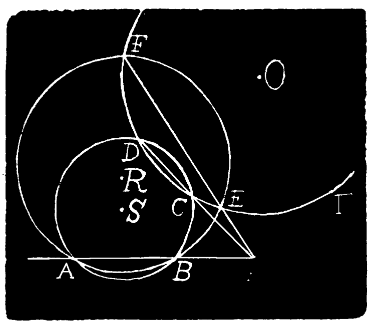
If a given circle is cut by another circle passing through two fixed points the common chord passes through a fixed point.

I. Solution by GEORGE R. DEAN, C. E., B. Sc., High School. Kansas City, Missouri.

The straight line containing the two given points is the radical axis of every pair of circles to which the points are common. Let the radical axis of the given circle and one of these circles intersect the given radical axis at some point O; then the radical axis of the given circle and any other circle containing the given points must pass through O, for the radical axis of three circles meet in a point.

II. Solution by J. C. GREGG, Superintendent of Schools, Brazil, Indiania; and P. S. BERG, Apple Creek, Ohio.

Let A and B be two fixed points and O the center of a fixed circle. Let R be the center of any circle through A and B and cutting circle O in D and C.



To show that the chord DC passes through a fixed point. Produce AB and DC to meet in P; then P is the required point. Draw the tangent PT. Then we have $PA.PB=PD.PC=\overline{PT}^2\ldots.(1)$.

Draw any other circle (center S) through A and B and cutting circle O, in two points one of which is E. Draw PE and produce it till it cuts circle S in some point X and O in F. Now from the secants PA and PX we have $PE.PX=PA.PB=\overline{PT}^2$ from (1) and from secant PF and tangent PT we have $PE.PF=\overline{PT}^2$. $\therefore$ $PE.PX=PE.PF$ and hence $PX=PF$ and the points X and F coincide and are the intersection of circles S and O